**Homework 13**

**P23.1.3** Determine the FT of the following functions:

(a) 

(b) , by direct evaluation and by using Equation 23.1.13.

(c) (Hint: use Equation 23.1.13).

**Solution:** (a) . From the given table of Fourier transform pairs, the FT of  is = *F*(*jω*).

(b) From the result of Exercise 23.1.1 and the differentiation property,

F{} =. To derive this directly, . Integrating by parts, = 0 +, as before. Note that , and remains finite as , whereas as .

(c) ; from Equation 23.1.1, the FT of is . Invoking duality, the FT of is = , so that the FT of is , and the FT of is . Hence *F*(*jω*) = +).

**P23.1.5** Determine the IFT of the following functions:

(a) 

(b) .

**Solution:** (a) The FT of is . It follows that the FT of . Hence, the IFT of  is .

(b) =; the IFT of  is ; the IFT of  is ; from the time reversal property, the IFT of , whose IFT is -; the IFT of  is ; it follows that the IFT of the given function is .

**P23.1.8** Determine *F*(*jω*) of *f*(*t*) in Figure P23.1.8, and verify the interpretation of *F*(0).

**Solution:** ; = .  = ; hence,    + , and .

To determine *F*(0), we differentiate the numerators and denominators of the first term once and of the second term twice and set *ω* = 0. This gives *F*(0) = *Aτ*/2, which is the area under the curve.

**P23.1.9** Determine *F*(*jω*) of *f*(1)(*t*), where *f*(*t*) is that in Figure P23.1.8, and verify the result by applying the differentiation-in-time property to the result of Problem P23.1.8. Compare to Equation 23.1.12.

**Solution:** *f*(1)(*t*) is the time derivative of *f*(*t*) as shown. The FTs of the impulses is = . The FT of the pulses is F = . This is the same result as in Example 23.1.1, with A replaced by 2*A*/*τ* is . Adding the FT of the impulses gives: . This is the same as the result of p23.1.8 multiplied by *jω*, in accordance with the differentiation-in-time property.

**P23.1.15** Assume that the function shown in Figure P23.1.8 is in the frequency domain. Determine *f*(*t*) and verify by applying duality to the result of P23.1.8.

**Solution:** ; 

= ;   = ; hence,  +  , and .

 Applying duality, *F*(*jt*) is obtained by substituting *t* for *ω* in *F*(*jω*) obtained in P23.1.8, which gives: . This is 2*πf*(-*ω*), where *f*(-*ω*) = *f*(*ω*), because the function is even, and *f*(*ω*) is the function of Figure P23.1.8 in the frequency domain. Thus, the IFT of *f*(*ω*) is , as before.

**P23.1.19** Determine *F*(*jω*) of *f*(*t*) in Figure P23.1.19, and express it in the simplest possible form.

**Solution:** In terms of step functions, *f*(*t*) = *u*(*t* + 4) – 2*u*(*t* + 3) + 2*u*(*t* + 1) – 2*u*(*t* – 1) + 2*u*(*t* – 3) – u(*t* – 4). *F*(*jω*) = = . At *ω* = 0, the *δ*(*ω*)sin terms are zero, so that *F*(*jω) =* , or in terms of a sinc function, *F*(*jω) =* 8sinc(4*ω*) – 12sinc(3*ω*) + 4sinc(*ω*).

Alternatively, *f*(1)(*t*) = *δ*(*t* + 4) – 2*δ*(*t* + 3) + 2*δ*(*t* + 1) – 2*δ*(*t* – 1) + 2*δ*(*t* – 3) – *δ*(*t* – 4). The FT is . *f*(1)(*t*) has zero average, so that the FT of its integral is obtained by dividing by *jω*, which gives the same expression as before.

**P23.1.24** Determine the inverse Fourier transform of the convolution of  and .

**Solution:** From the table of Fourier transform pairs,  is the FT of , and is the FT of . From the convolution-in-frequency property, the required IFT is = .

**P23.1.29** Determine *F*(*jω*) of the periodic triangular waveform of Figure 16.3.4 and verify it by applying the integration-in-time property to the square wave of Example 23.3.2.

**Solution:** From Equation 16.3.18, odd. It follows that , *n* odd. For the square wave, , where . Substituting and applying the integration-in-time property, the *n*th term becomes , as above.

